

# AN ANALYSIS OF THE EFFECT OF ELONGATIONAL VISCOSITY ON THE FLOW IN A FLAT DIE

Y. Sun and M. Gupta

Mechanical Engineering-Engineering Mechanics Department  
Michigan Technological University  
Houghton, MI 49931

## Abstract

The flow of a low-density polyethylene in a flat die is simulated using the axisymmetric and planar elongational viscosities estimated in an earlier publication by Beaupre and Gupta. Elongational viscosity is found to have only a limited effect on the velocity distribution at the die exit. However, the predicted pressure drop in the die and temperature distribution at the die exit changed significantly when the effect of elongational viscosity is included in the simulation.

## Introduction

Screw extruders [1] are used to manufacture plastic parts for a large number of end-use applications such as pipes, sidings for building, seals for car windows, etc. Even though the same extruder can be used to manufacture many of these plastic parts, a different die is required for each product. The die determines the dimensions and quality of the extruded products. Therefore, use of an optimally designed die is crucial for the quality of the extruded parts. The main task in design of an extrusion die is to optimize the die channel geometry such that uniform velocity and temperature distributions are obtained at the die exit, without excessively increasing the pressure drop in the die [2]. This requires, an analysis of the velocity, pressure and temperature fields in the polymeric flow in the die. However, at present, most die designs for polymer extrusion are based upon the past experience of the designer, trial and error, or at most a flow simulation using the generalized Newtonian formulation [3 – 6]. A generalized Newtonian constitutive equation can accurately capture the strain-rate dependence of the shear viscosity of a polymer. However, the elongational viscosity predicted by a generalized Newtonian formulation is generally much smaller than the actual elongational viscosity of a polymer. Therefore, in applications involving elongation-dominated flows, such as extrusion dies, the velocity, pressure and temperature fields predicted by the generalized Newtonian formulation can have large errors. Even though some of the viscoelastic constitutive equations [7] can predict the high elongational viscosity of polymers, difficulty in convergence of viscoelastic flow simulations, along with the large computation time required for the simulation, and difficulty in finding the values of various parameters

in the constitutive equations has limited the application of viscoelastic flow simulation in design of extrusion dies.

In the present work, the PELDOM software [8] was used for a three-dimensional simulation of the flow in a flat extrusion die. To simulate a polymeric flow, the software accounts for strain-rate dependence of the shear as well as the elongational viscosity of the polymer. For a three-dimensional simulation of a polymeric flow, the software requires knowledge of the axisymmetric and planar elongational viscosities of the polymer. The method used in the present work for estimating the elongational viscosity of a polymer is discussed next.

## Elongational Viscosity Estimation

In the present work, the axisymmetric and planar elongational viscosities of a low-density polyethylene (Dow 132i) estimated by Beaupre and Gupta [9] were used to investigate the effect of elongational viscosity on the flow in a flat die. Beaupre and Gupta [9] used the entrance-flow method to estimate the axisymmetric and planar elongational viscosities of the low-density polyethylene (LDPE). In the entrance-flow method, the large pressure drop (entrance loss) encountered near an abrupt contraction in a channel (entrance flow), which increases as the elongational viscosity of the fluid is increased, and also depends on the flow rate in the channel, is used for an indirect measurement of the strain-rate dependence of the elongational viscosity of a polymer. In particular, Beaupre and Gupta [9] optimized the value of the four elongational viscosity parameters ( $\delta, \lambda_1, \lambda_2, m$ ) in the Sarkar-Gupta elongational viscosity model [10] (Eqn. 1) such that the difference between the experimental value of the entrance pressure loss from a capillary or slit rheometer and the corresponding predictions from a finite element simulation is minimized.

$$\eta_e = \eta_0 \left[ T_r + \delta \left\{ 1 - \frac{1}{\sqrt{1 + (\lambda_1 e_{II})^2}} \right\} \right] (1 + (\lambda_2 e_{II})^2)^{\frac{m-1}{2}} \quad (1)$$

where  $e_{II} = \sqrt{2(\tilde{\epsilon} : \tilde{\epsilon})}$  is the second invariant of the strain-rate tensor  $\tilde{\epsilon} = (\nabla \hat{v} + \nabla \hat{v}^T) / 2$ , with  $\hat{v}$  being the velocity of the fluid,  $T_r$ , Trouton ratio at low shear rate, is 3 for an axisymmetric flow and 4 for a planar flow, and

$\eta_0$ ,  $\delta$ ,  $\lambda_1$ ,  $\lambda_2$  and  $m$  are material parameters. To capture the strain-rate dependence of the shear viscosity of a polymer, Carreau model [11] is used in this work:

$$\eta_s = \eta_0 (1 + (\lambda e_{II})^2)^{\frac{n-1}{2}} \quad (2)$$

where  $\eta_0$ , the zero-shear viscosity, is the same as in Eqn. (1), and  $\lambda$  and  $n$  are the two new material parameters. The Arrhenius model [11] is used here to capture the temperature dependence of  $\eta_0$  in Eqns. (1) and (2):

$$\eta_0 = A \exp\left(\frac{T_a}{T}\right) \quad (3)$$

where  $A$  and  $T_a$  are material parameters, and  $T$  is the temperature of the polymer.

For Dow 132i, Beaupre and Gupta [9] gave the Carreau model parameters for shear viscosity at 215 and 230 °C. The values of the Carreau model parameters given by Beaupre and Gupta [9] were used in the present work to obtain the parameters for the Carreau-Arrhenius model for LDPE:

$$A = 469.0 \text{ Pa} \cdot \text{s}, \quad T_a = 2721.7 \text{ K} \\ \lambda = 0.6509 \text{ s}, \quad n = 0.401.$$

By using the entrance-flow method for elongational viscosity estimation, for the LDPE at 215 and 230 °C, Beaupre and Gupta [9] also gave the values of elongational viscosity parameters for the Sarkar-Gupta model. These values from reference [9] were used in the present work to obtain the temperature-dependence of Sarkar-Gupta model for axisymmetric and planar elongational viscosities.

Axisymmetric:  $\delta = 0$ ,  $\lambda_2 = 0.02242 \text{ s}$ ,  $m = 0.349$ .

Planar:  $\delta = 37.3$ ,  $\lambda_1 = 11.795 \text{ s}$ ,  $\lambda_2 = 0.4571 \text{ s}$ ,  $m = 0.45$ .

It should be noted that  $\lambda$  in the Carreau model for shear viscosity, and  $\lambda_1$  and  $\lambda_2$  in the Sarkar-Gupta model for elongational viscosity depend upon temperature. Temperature dependence of  $\lambda$ ,  $\lambda_1$  and  $\lambda_2$  can be easily obtained by using the time-temperature superposition along with the Arrhenius model (Eqn. 3).

Based upon the values of the parameters for the Carreau model and the Sarkar-Gupta model given above, the shear and elongational viscosities of the LDPE at 215 and 230 °C are shown in Fig. 1. Fig. 1 also shows the elongational viscosity predicted by Cogswell's [13] and Binding's [14] analyses. In general, the elongational viscosity estimated by the Cogswell's and Binding's analyses are higher than the corresponding estimations by optimizing the values of elongational viscosity parameters in the Sarkar-Gupta model. The discrepancy is particularly large for the planar elongational viscosity. The predictions from Cogswell's analysis in Fig. 1 show that the average elongation rate for the experimental data on planar entrance loss is in the transition region between elongation thickening and elongation thinning portions of the elongational viscosity curve, which might have resulted in

the large difference between the prediction from the Cogswell's and Binding's analyses and those from the finite element simulation by Beaupre and Gupta [9]. Since the experimental data for the axisymmetric entrance flow is in the power-law region of the elongational viscosity, the predictions of Beaupre and Gupta are in better agreement with those from the Cogswell's and Binding's analyses.

## Effect of Elongational Viscosity on the Flow in a Flat Die

In this section, the shear and the axisymmetric and planar elongational viscosities for LDPE, given in the last section, are used for a three-dimensional simulation of the flow in a flat extrusion die. Fig. 2 shows the geometry and the finite element mesh for the flat die used in this work. The finite element mesh has 35,375 nodes and 166,090 elements. The geometric parameters for the flat die in Fig. 2, which are taken from reference [6], are given in Table 1. The die has a relatively small primary manifold and a long secondary manifold before the land near the die exit. A triangular preland section is located between the primary and secondary manifolds. The extra pressure drop near the middle of the triangular preland compensates the non-uniform pressure in the primary manifold which decreases with distance from the feed point. The tetrahedral finite elements employed in this work, use a linear interpolation for pressure and temperature, whereas a linear interpolation enriched with a piecewise-linear bubble function is used for the velocity interpolation over tetrahedral finite elements [15]. However, the velocity equations corresponding to the bubble node are eliminated at the element level using static condensation [16], resulting in a highly efficient finite element simulation.

Fig. 3 compares the velocity distributions in the mid-plane of the flat die predicted by the generalized Newtonian formulation (Fig. 3 a) and by including the effect of elongational viscosity on the flow (Fig. 3 b). The velocity distributions in Figs. 3 (a) and (b) have some minor differences only. The velocity distribution at the exit of the flat die predicted by including the effect of elongational viscosity (Fig. 3 b) is slightly more uniform. Accordingly, for the same flow rate in the die, the maximum velocity in Fig. 3 (b) (0.304 m/s) is slightly lower than that in Fig. 3 (a) (0.306 m/s).

The goal in design of a flat die is to obtain a uniform thickness across the width of the extruded sheet. The velocity and temperature distributions across the width of the die exit can significantly affect the thickness of the sheet. The sheet is generally thicker at the locations with higher velocity at the die exit. Accordingly, in comparison to the sheet thickness near the two ends, the LDPE sheet extruded by the flat die in Fig. 2 is expected to have a slightly larger thickness near the center. Except at the small regions near the ends, the predicted velocity in most of the die exit is quite uniform. Therefore, the die in Fig. 2 is well

balanced for LDPE. Since the velocity distributions in Fig. 3 (a) and (b) are very similar, it may be expected that the elongational viscosity of LDPE has only a minor effect on the thickness of the extruded sheet. However, the temperature of the plastic at the die exit also affects the final sheet thickness. If the velocity across the die exit is uniform, because of the higher shrinkage during cooling of the sheet, thickness is expected to be smaller at the locations with higher temperature. Fig. 4 compares the temperature distributions in the mid plane of the flat die predicted by a generalized Newtonian formulation (Fig. 4 a) and by including the effect of elongational viscosity (Fig. 4 b). The temperature distributions at the die exit in Figs. 4 (a) and (b) are significantly different, with the temperature at many locations along the die exit in Fig. 4 (b) about 8 degrees higher than that in Fig. 4 (a). The difference in the temperature in Figs. 4 (a) and (b) can affect the final thickness of the extruded sheet. It is noted that the main difference in the temperature distributions in Figs. 4 (a) and (b) is in the land near the exit of the die. Away from the exit, the temperature distributions in Figs. 4 (a) and (b) have some minor differences only. In the die shown in Fig. 2, just before the land, there is a large reduction in the die thickness. The flow in this contraction region before the land is highly elongation dominated. In the plane of symmetry parallel to the thickness direction, Fig. 5 shows the temperature distribution in this elongation-dominated region near the exit of the flat die. It is evident in Fig. 5 that the temperature of polymer predicted in the elongation-dominated region increases significantly as the effect of elongational viscosity is included in the simulation. With the effect of elongational viscosity included in the simulation, extra work is done to push the polymer through the elongation-dominated region. This extra work is converted into heat resulting in the higher temperature in the converging region before the land. In other words, in elongation-dominated flows the heat generated due to viscous dissipation strongly depends on the elongational viscosity of the polymer. This extra heat generated due to the high elongational viscosity of the polymer, which is being captured by the flow simulation, is convected along the die land, resulting in the higher temperature at the exit in Figs. 4 (b) and 5 (b).

Fig. 6 compares the predicted pressure distributions in the flat die by a generalized Newtonian formulation (Fig. 6 a) and by including the effect of elongational viscosity on the flow (Fig. 6 b). When the effect of elongational viscosity is included in the simulation (Fig. 6 b), the predicted pressure drop across the die is about 17% larger than the pressure drop predicted by the generalized Newtonian formulation (Fig. 6 a). The larger pressure in Fig. 6 (b) implies that in a screw extruder, if a constant pressure is available at the die entrance, the actual flow rate in the die will be smaller than the flow rate predicted by a flow simulation using the generalized Newtonian formulation.

## Conclusions

The axisymmetric and planar elongational viscosities of a low-density polyethylene, estimated by Beaupre and Gupta [9] using entrance-flow method, were used in the present work to simulate the flow in a flat die. For the flow of LDPE in the flat die used, the elongational viscosity is found to have only a minor effect on the velocity at the die exit. However, the predicted pressure drop in the die and temperature distribution at the die exit changed significantly when the effect of elongational viscosity was included in the simulation.

## Acknowledgement

This work is supported by the National Science Foundation Grant DMI-0200091.

## References

1. Z. Tadmor and I. Klien, *Engineering Principles of Plasticating Extrusion*, Van Nostrand Reinhold, New York (1970).
2. W. Michaeli, *Extrusion Dies for Plastics and Rubber*, Hanser Publishers, New York (1992).
3. M. Gupta, Y. Jaluria, V. Sernas, M. Esseghir and T. H. Kwon, *Poly. Eng. Sci.*, **33**, 393 (1993).
4. R. Shankar and R. Ramanathan, *SPE ANTEC Tech. Papers*, **41**, 65 (1995).
5. W. A. Gifford, *SPE ANTEC Tech. Papers*, **44**, 290 (1998).
6. W. A. Gifford, *Poly. Eng. Sci.*, **41**, 1886 (2001).
7. R. G. Larson, *Constitutive Equations for Polymeric Melts and Solutions*, Butterworths, Boston (1988).
8. PELDOM software, Plastic Flow, LLC, 1206 Birch Street, Houghton, MI 49931 ([www.plasticflow.com](http://www.plasticflow.com)).
9. P. Beaupre and M. Gupta, *SPE ANTEC Tech. Papers*, **48**, 892 (2002).
10. D. Sarkar and M. Gupta, *J. Reinf. Plast. Comp.*, **20**, 1473 (2001).
11. R. B. Bird, R. C. Armstrong and O. Hassager, *Dynamics of Polymeric Liquids*, Vol. 1 and 2, Wiley, New York (1987).
12. M. Gupta, *Adv. Poly. Technology*, **21**, 98 (2002).
13. F. N. Cogswell, *Poly. Eng. Sci.*, **12**, 64 (1972).
14. D. M. Binding, *J. Non-Newt. Fluid Mech.*, **27**, 193 (1988).
15. E. Pichelin and T. Coupez, *Comput. Mech. Eng.*, **163**, 359 (1998).
16. K. J. Bathe, *Finite Element Procedures*, Prentice Hall, New Jersey (1996).

Table 1: Geometric parameters [6] for the flat die in Fig. 2.

Lengths		cm (inches)
Die width		101.6 (40)
Die length (including channel)		33.0 (13)
Inlet width		10.16 (4)
Manifold center depth		3.81 (1.5)
Manifold side depth		1.02 (0.4)
Manifold center flat length (including radius)		1.52 (0.6)
Manifold side flat length (including radius)		1.02 (0.4)
Manifold end sweep length		10.16 (4)
Preland gap		0.61 (0.24)
Preland center length		5.08 (2)
Preland side length		0.00254 (0.001)
Secondary manifold center length		5.08 (2)
Secondary manifold side length		5.08 (2)
Secondary manifold depth		0.635 (0.25)
Land length		2.54 (1)
Land gap in center		0.102 (0.04)
Angles		Degrees
Manifold backwall angle		85°
Manifold angle		20°
Secondary manifold angle		20°

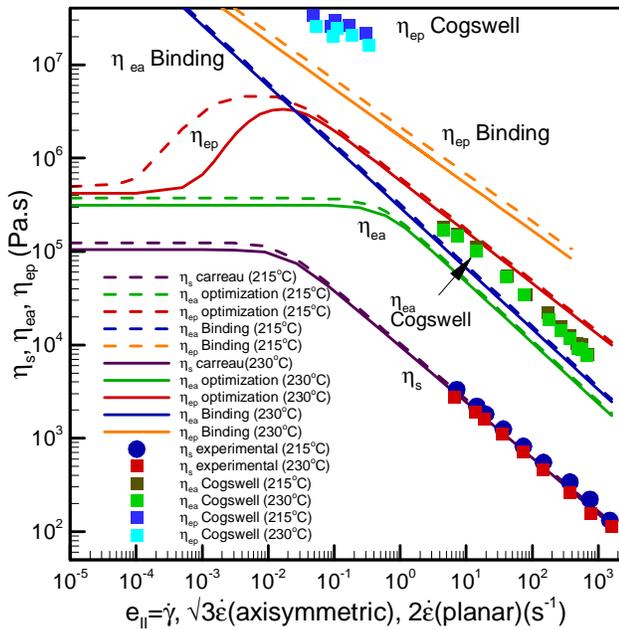


Fig. 1 Variation of shear ( $\eta_s$ ) and elongational ( $\eta_{ep}$  (planar),  $\eta_{ea}$  (axisymmetric)) viscosities of LDPE with the second invariant of strain-rate tensor ( $e_{II}$ ).

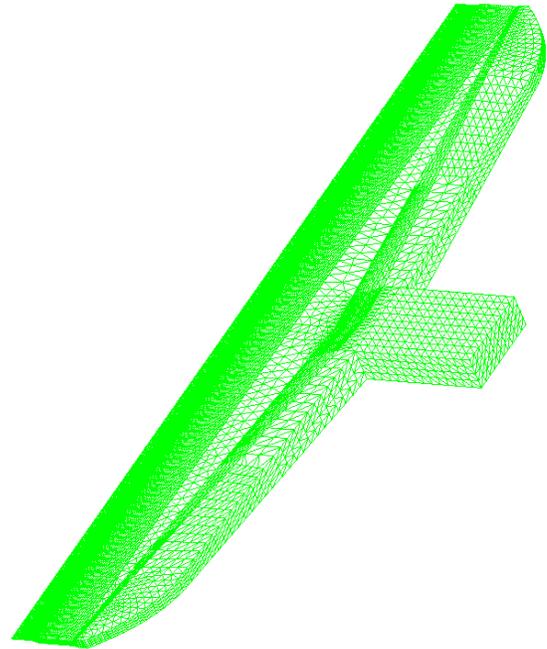


Fig. 2 Finite element mesh in the flat die.

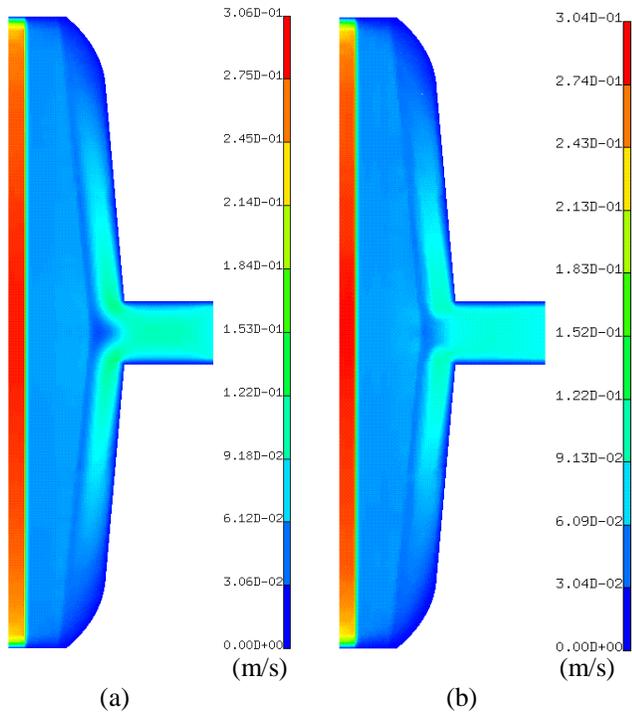


Fig. 3 Velocity distribution in the mid-plane of the flat die. (a) Carreau model with generalized Newtonian formulation, (b) Carreau model for shear viscosity along with Sarkar-Gupta model for elongational viscosity.

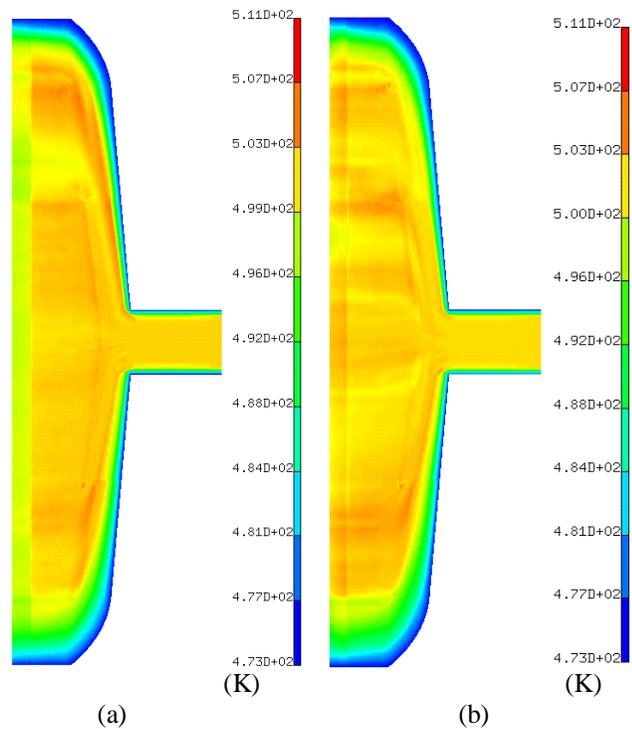


Fig. 4 Temperature distribution in the mid-plane of the flat die. (a) Carreau model with generalized Newtonian formulation, (b) Carreau model for shear viscosity along with Sarkar-Gupta model for elongational viscosity.

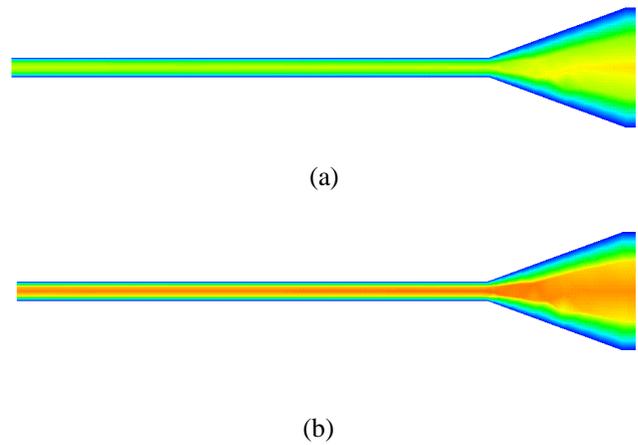


Fig. 5 Temperature distribution in the mid-plane parallel to the thickness direction of the flat die. (a) Carreau model with generalized Newtonian formulation, (b) Carreau model for shear viscosity along with Sarkar-Gupta model for elongational viscosity.

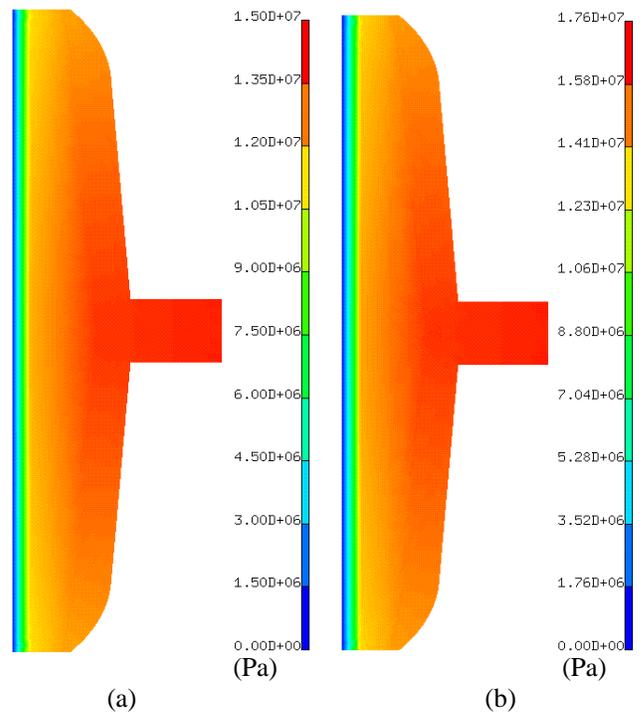


Fig. 6 Pressure distribution in the mid-plane of the flat die. (a) Carreau model with generalized Newtonian formulation, (b) Carreau model for shear viscosity along with Sarkar-Gupta model for elongational viscosity.