EFFECT OF ELONGATIONAL VISCOSITY ON DIE DESIGN FOR PLASTIC EXTRUSION

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Abstract

A finite element simulation of the flow in a rectangular die with an abrupt contraction is presented. Effect of shear and elongational viscosities of a polymer on the flow in the die is analyzed. The shear and elongational viscosities are represented by a truncated power-law model. The power-law indices for the planar and axisymmetric elongational viscosities are found to have significant effect on the pressure and velocity fields in the die.

Introduction

Extrusion die determines the dimensions and quality of the plastic extudate from a screw extruder. Therefore, use of a well-designed extrusion die is essential to assure the quality of the extruded product. Design of an extrusion die requires an analysis of the velocity as well as pressure field for the polymeric flow inside the die [1]. For the desired throughput, the pressure required at the die entrance should not exceed the pressure available at the screw channel exit, which depends upon the screw speed. Therefore, at a constant screw speed, an excessively restrictive flow in the die leads to a reduction in the throughput. Knowledge of the pressure field in a die is also important for a robust mechanical design of the die such that the internal pressure does not cause any significant change in the flow channel dimensions. The velocity field in an extrusion die should be such that the average flow rate for any portion of the outlet cross-section is the same. Also, to avoid stagnation and decomposition of the extruded material, the velocity field should not have any recirculating vortices.

To simulate the flow in extrusion dies, a generalized Newtonian constitutive equation has generally been used in the past [2-4]. A generalized Newtonian constitutive equation can accurately capture the shear-thinning behavior of the viscosity of a polymer, however, the elongational viscosity predicted by a generalized Newtonian model can be quite different from that of the polymer. Therefore, in applications involving a significant elongational flow, such as extrusion dies, the velocity and pressure fields predicted by a generalized Newtonian formulation can have large error. An accurate simulation of the flow in an extrusion die requires a constitutive equation which can predict the shear as well as elongational viscosity of a polymer. Even though many of the viscoelastic constitutive equations can predict the high elongational viscosity of polymers [5], difficulty in convergence of viscoelastic flow simulation at high flow rates [6-10] and that of finding the values of various parameters in the constitutive equation has somewhat limited the application of viscoelastic flow simulation.

In the present work, a software for three-dimensional simulation of polymeric flows has been developed. To simulate a polymeric flow, this software accounts for the strain-rate dependence of shear as well as elongational viscosities of the polymer. This newly developed software has been used in this paper to simulate the flow in a rectangular die with abrupt contraction. Effect of elongational viscosity on recirculating vortices and pressure drop in the rectangular die is analyzed in this paper.

Shear and Elongational Viscosities of a Polymer

The shear and elongational viscosities of a fluid are defined as follows [11] :

$$\tau_{12} = \eta_s \dot{\gamma} \tag{1}$$

$$\tau_{11} - \tau_{22} = \eta_e \dot{\varepsilon} \tag{2}$$

where τ_{--} denotes various components of the stress tensor, η_s and η_e are respectively, the shear and elongational viscosities, with $\dot{\gamma}$ and $\dot{\varepsilon}$ being the corresponding strain rates.

For a Newtonian fluid, the shear and elongational viscosities are independent of the strain rate. However the elongational viscosity depends upon the elongation mode defined by the parameter b in Eqn. (3) below. For a general shear-free flow, the strain-rate tensor ($\tilde{e} = (\nabla\hat{v} + \nabla\hat{v}^T)/2$), where $\hat{v}$ is the velocity, can be written as follows:

$$\tilde{e} = \dot{\varepsilon}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -(b+1)/2 & 0 \\ 0 & 0 & (b-1)/2 \end{pmatrix} \tag{3}$$

For $b = 0$, which corresponds to an axisymmetric elongational flow, the shear and elongational viscosities of a Newtonian fluid satisfy the Trouton equation [11],

$$\eta_a = 3\eta_s, \tag{4}$$

whereas, for a planar elongational flow ($b = 1$), the corresponding equation is,

$$\eta_p = 4\eta_s, \quad (5)$$

where η_a and η_p are the elongational viscosities for the axisymmetric and planar cases, respectively.

The shear and elongational viscosities of a polymer depend upon strain rate. At low values of strain rate, shear and elongational viscosities of a polymer are constant. The zero-strain-rate viscosities of a polymer also satisfy the Trouton equations for axisymmetric and planar cases (Eqns. 4 and 5) [11]. Beyond the Newtonian limit, shear viscosity of a polymer decreases with increasing strain rate, whereas, for most polymers the elongational viscosity increases with strain rate, which is followed by a descent as the strain rate is further increased [12].

For generalized Newtonian fluids, shear viscosity is represented as a function of the second invariant of the strain rate tensor,

$$\eta_s = \eta_s(e_{II}),$$

where $e_{II} = \sqrt{2(\tilde{e}:\tilde{e})}$. In the present work, the axisymmetric and planar elongational viscosities have also been represented as functions of e_{II} as defined above. It should be noted that for a simple shear flow $e_{II} = \dot{\gamma}$, whereas for an elongational flow, for the axisymmetric case $e_{II} = \sqrt{3}\,\dot{\varepsilon}$ and for the planar case $e_{II} = 2\dot{\varepsilon}$. In the literature, η_a and η_p are generally specified as functions of $\dot{\varepsilon}$ and not that of e_{II}. The approach used in this paper has unified the independent variable for the shear and elongational viscosities. Furthermore, with e_{II} as the independent variable, irrespective of the strain rate, the Trouton equations for the axisymmetric and planar cases (Eqns. 4 and 5) are satisfied for all generalized Newtonian constitutive equations.

In this paper, the shear and elongational viscosities of a polymer have been represented by the truncated power-law model (Fig. 1),

$$\eta_s = Ae_{II}^{n-1} \text{ for } e_{II} > e_0,\ \eta_s = \eta_0 \text{ for } e_{II} \le e_0, \quad (6)$$

$$\eta_a = Be_{II}^{m_a-1} \text{ for } e_{II} > e_0,\ \eta_a = 3\eta_0 \text{ for } e_{II} \le e_0, \quad (7)$$

and

$$\eta_p = Ce_{II}^{m_p-1} \text{ for } e_{II} > e_0,\ \eta_p = 4\eta_0 \text{ for } e_{II} \le e_0. \quad (8)$$

The power-law indices for the shear (n) and elongational (m_a, m_p) viscosities can be different. It is noted that for $m_a = m_p = n$, the model given by Eqns. 6 - 8 is identical to a purely viscous generalized Newtonian formulation for a truncated power-law model with power-law index of n. Effect of elongational power-law indices (m_a, m_p) on the flow in a rectangular die is analyzed later in the paper.

Flow in a Rectangular Die with Abrupt Contraction

Fig. 2 shows the finite element mesh used to simulate the flow in a rectangular die with abrupt contraction. The lengths of the sides of the square cross-section upstream and downstream of the contraction are 8*b* and 2*b*, respectively. Lengths of the upstream and downstream channels are respectively, 10*b* and 20*b*. Using the symmetry of the problem, only the flow in a quarter of the die was simulated. Along with the symmetry condition on the two planes of symmetry and no-slip condition on the walls, average velocity for the required flow rate was enforced at the entrance. For a fully developed flow at the exit, two velocity components perpendicular to the die axis and the traction force along the axis were specified to be zero. A fourteen-noded brick element ($Q_1^+P_0$) with velocity nodes at the eight corners and at the centers of the six faces [13] was used for the flow simulation. Pressure is constant within the $Q_1^+P_0$ element and the velocity nodes at the center of the faces have only one degree of freedom, which is the velocity component normal to the face [13].

For the power-law index for shear viscosity $n = 0.25$, Newtonian limit $e_0 = 0.001\ \mathrm{s}^{-1}$ and the average strain rate $\dot{\gamma}_a = U / b = 1\ \mathrm{s}^{-1}$, where U is the average velocity in the downstream channel, Fig. 3 shows the effect of elongational power-law indices on the recirculating vortices in the rectangular die. To depict the flow direction, the unit vectors in the direction of velocity are shown in Fig. 3. Flow in a plane of symmetry (left column in Fig. 3) as well as the flow in a plane passing through a diagonal of the square cross-section (right column in Fig. 3) is shown. For $n = m_a = m_p = 0.25$, which corresponds to a purely viscous generalized Newtonian formulation for the truncated power-law model with $n = 0.25$, no recirculating vortex is formed. Even for $m_a = m_p = 0.5$, no significant recirculation is found in the plane of symmetry or diagonal plane. As the elongational power-law indices, m_a and m_p, are increased to 0.7, recirculating vortices are observed in the plane of symmetry as well as in the diagonal plane. In comparison to the vortex in the plane of symmetry, a larger vortex is formed in the diagonal plane. A further increase in the value of the elongational power-law indices, m_a and m_p, to 0.8 results in a significant growth in the recirculating vortices. Even for $m_a = m_p = 0.8$ the recirculating vortex in the diagonal plane is larger than the vortex in the plane of symmetry.

For the four cases presented in Fig. 3, the power-law indices for the axisymmetric and planar elongational viscosities were the same. Fig. 4 shows the flow direction for the two cases with different values of m_a and m_p. For $n = 0.25$, $m_a = 0.8$ and $m_p = 0.5$, no significant recirculation is found in the plane of symmetry or the diagonal plane, whereas for $n = 0.25$, $m_a = 0.5$ and $m_p = 0.8$, large recirculating vortices are formed in the plane of symmetry as well as in the diagonal plane. Recirculating vortices for $n = 0.25$, $m_a = 0.5$ and $m_p = 0.8$ are even larger than the vortices shown in Fig. 3 (d) for $n = 0.25$, $m_a = m_p = 0.8$.

For various values of elongational power-law indices, Fig. 5 shows the velocity along the axis of symmetry. Since the average velocity for the required flow rate was enforced at all the entrance nodes, the centre-line velocity increases near the entrance. For $n = m_a = m_p = 0.25$, which corresponds to the truncated power-law model for a purely viscous generalized Newtonian fluid with power-law index $n = 0.25$, near the abrupt contraction the centre-line velocity overshoots its value for a fully developed flow. This kink in the centre-line velocity is maintained as the elongational power-law indices are increased. However, at higher values of elongational power-law indices, the kink is observed at a lower velocity and a larger distance is required for the flow to develop completely.

Pressure variation along the axis of symmetry for different values of elongational power-law indices are shown in Fig. 6. Since the flow near the abrupt contraction is highly elongation dominated, besides the pressure drop for a fully developed flow in the upstream and downstream channels, an additional pressure loss is encountered near the abrupt contraction in the rectangular die. This steep drop in the pressure near the abrupt contraction increases sharply as the power-law indices for the axisymmetric and planar elongational viscosities are increased.

Conclusions

A finite element software for a three-dimensional simulation of polymeric flows has been developed. The shear and elongational viscosities of a polymer have been represented by the truncated power-law model with different power-law indices for the shear and elongational viscosities. For a constant value of shear viscosity parameters, the effect of power-law indices for axisymmetric and planar elongational viscosities on the recirculating vortices and pressure loss in a rectangular die was analyzed. Extra pressure loss due to the elongational flow near the abrupt contraction in the die was found to increase rapidly with the elongational power-law indices.

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Key words: Elongational viscosity, Extensional viscosity, Extrusion, Die.

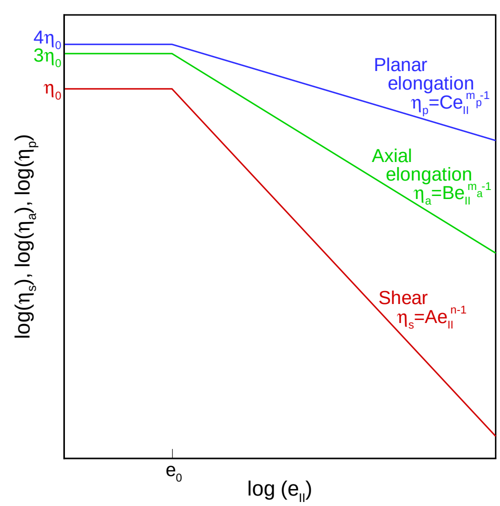

Fig. 1. Truncated power-law model for shear and elongational viscosities.

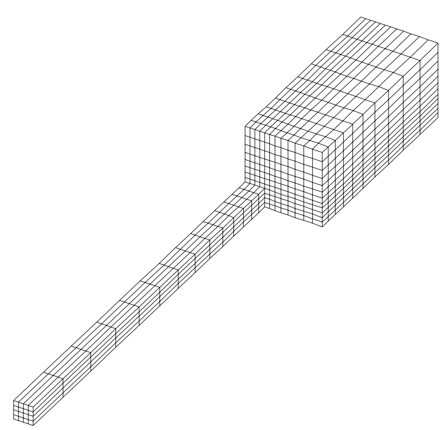

Fig. 2. Finite element mesh used for simulating the flow in a rectangular die with abrupt contraction.

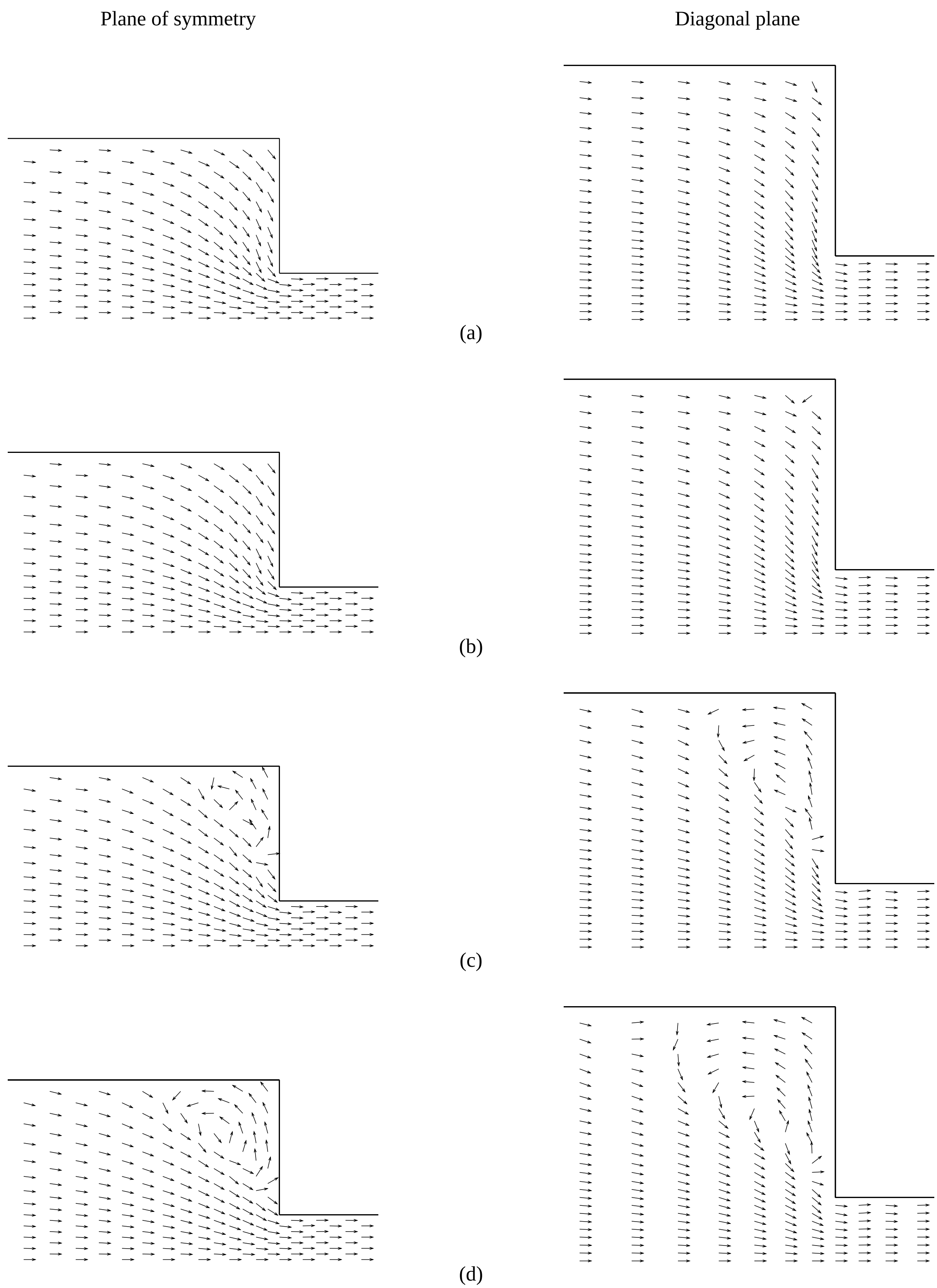

Fig. 3. Recirculation in the rectangular die for $n = 0.25$, $e_0 = 0.001\ s^{-1}$ and $\dot{\gamma}_a = 1\ s^{-1}$. For each of the four cases, the power-law indices for axisymmetric (m_a) and planar (m_p) elongational viscosities are the same. Values of m_a and m_p are (a) 0.25, (b) 0.5, (c) 0.7, (d) 0.8.

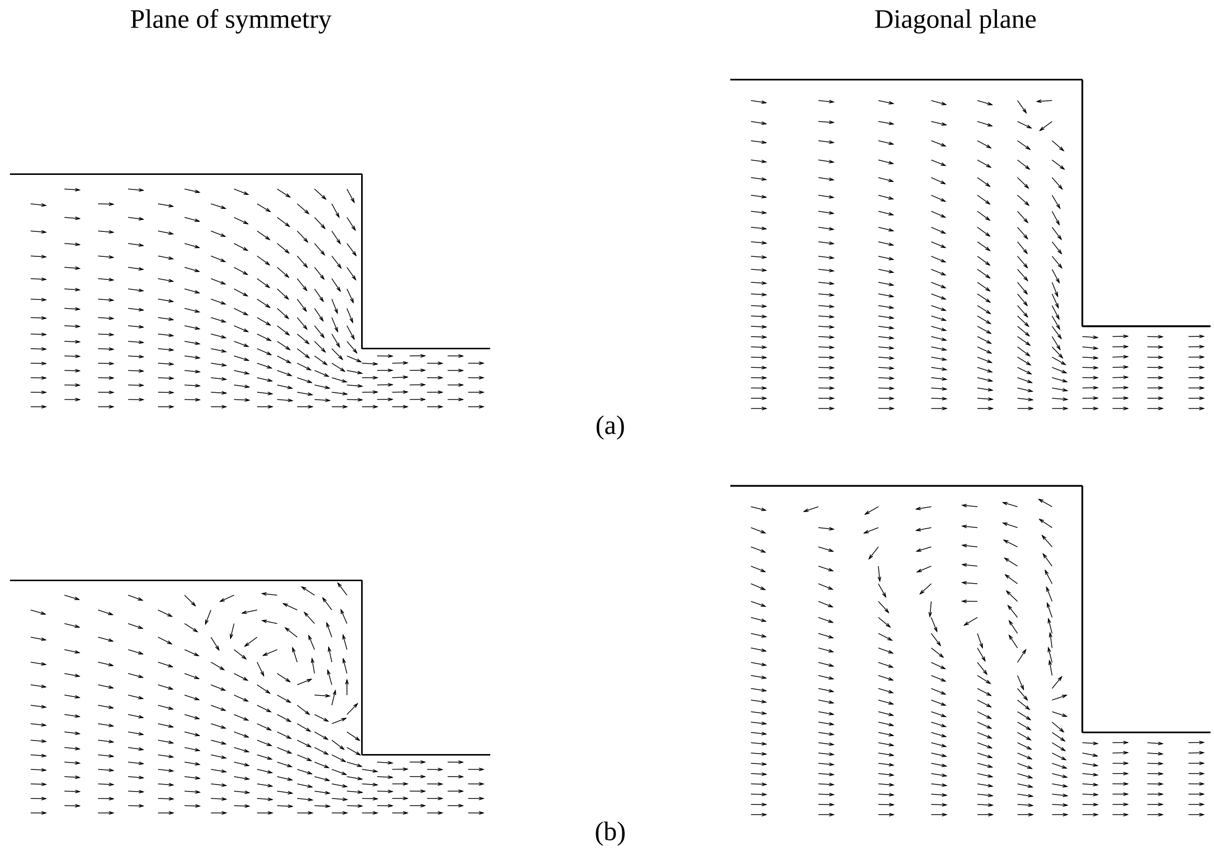

Fig. 4. Recirculation in the rectangular die for n = 0.25, e_0 = 0.001 s^{-1} and $\dot{\gamma}_a$ = 1 s^{-1}. The elongational power-law indices are (a) m_a = 0.8, m_p = 0.5, (b) m_a = 0.5, m_p = 0.8.

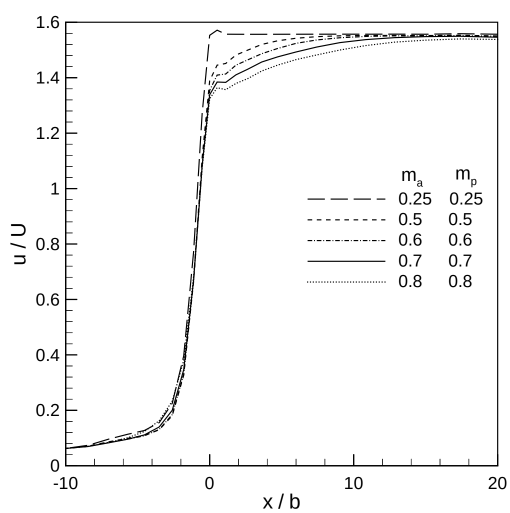

Fig. 5. Velocity along the axis of symmetry for n = 0.25, e_0 = 0.001 s^{-1}, $\dot{\gamma}_a$ = 1 s^{-1} and various values of the elongational power-law indices.

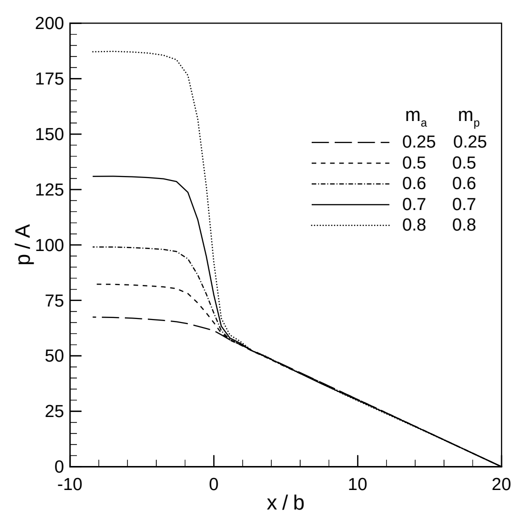

Fig. 6. Pressure along the axis of symmetry for n = 0.25, e_0 = 0.001 s^{-1}, $\dot{\gamma}_a$ = 1 s^{-1} and various values of the elongational power-law indices.